

Coherent reaction of Fermi superfluid on correlated disorder

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Motivated by the disparity between the experimentally observed properties of superfluid ^3He in aerogel and predictions of the Abrikosov and Gorkov theory of superconducting alloys we consider effect of correlated pair-breaking impurities on the temperature dependence of the square of magnitude of the average order parameter and related thermodynamic properties of a Fermi superfluid. We show that the correlations, increasing the spectral density of the fluctuations with small wave vectors increase the transition temperature and enhance a temperature region below T_c where fluctuations of the order parameter dominate. Outside of this region we treat effect of impurities as a perturbation and express corrections to thermodynamic properties in terms of the structure factor of impurities. Assuming a simple model expression for correlation function of aerogel we find corrections to the temperature dependence of the NMR frequency shifts for A-like and B-like phases of ^3He and compare these with experimental data.

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I. INTRODUCTION

Effect of impurities on traditional superconductors is well understood within the Abrikosov and Gor'kov (AG) theory of superconducting alloys.¹ For some of more recent superconducting and superfluid Fermi systems, including high- T_c compounds and superfluid ^3He , application of this theory is not always justified. Superfluid ^3He in high porosity silica aerogel^{2,3} is a good example of such situation. Thermodynamic properties of this object show both quantitative and qualitative discrepancies with the generalization of the AG theory for the p-wave Cooper pairing.⁴ The observed difference between the temperature of superfluid transition in the bulk liquid and in aerogel $T_{cb} - T_{ca}$ at pressures above 20 bar is about a half of the value, predicted by the theory.⁵ As an example of qualitative discrepancy we can cite the anomaly in the temperature dependence of the superfluid density and of the magnitude of transverse NMR frequency shift. In Ginzburg and Landau region both quantities instead of the linear growth with $T_{ca} - T$ demonstrate the dependence which can be approximated by a power law $(T_{ca} - T)^n$ with $n \approx 1.3 - 1.4$ and this dependence extends well below the T_c .⁶

The model calculations of the structure of aerogel show that it is an object, intermediate between the ensemble of conventional impurities and a porous media.^{7,8} On the one hand the scattering centers having diameter $d \approx 3$ nm are much smaller than the coherence length in ^3He ($\xi_0 \approx 20 \div 80$ nm), on the other – the structure has low density regions (“voids”) with the characteristic size of the order of ξ_0 . For improvement of the agreement with experiment more involved models were suggested which combine elements of description in terms of pores with that of impurities – isotropic inhomogeneous scattering model (IISM),⁹ and its simplified version – phenomenological IISM¹⁰, which is an interpolation between the two limits. The models contain additional parameters

(only one in the simplified version). The use of these as fitting parameters renders a satisfactory agreement between the calculated properties of ^3He and experimental data.⁵ Still these models have certain shortcomings on a theoretical side. The additional parameters are introduced on an intuitive basis without precise definition of their relation to the structure of aerogel. As a result the values of these parameters can not be found independently. It is not clear also how accurately predictions, based on these models describe effect of real aerogel on ^3He .

In the present paper we also take into account inhomogeneity of aerogel. Structural fluctuations, i.e. random deviations of the local density of impurities from the average density are treated as a perturbation. We concentrate on the effect of these fluctuations on thermodynamic properties of the superfluid ^3He near the T_c . A starting approximation is the AG theory, where impurities are treated as a continuous background, producing damping of quasiparticles and characterized by one parameter – mean free path of quasiparticles l . The first correction to the AG approximation can be expressed in terms of the structure factor of aerogel, which has precise definition and can be measured directly. The obtained results are approximate, but their accuracy is controlled by the small parameter, which is a combination of parameters, characterizing both aerogel and superfluid ^3He .

For account of structural fluctuations of pair breaking impurities we follow the argument of Larkin and Ovchinnikov.¹¹ They have shown that on a low temperature side of the transition $\langle \psi \rangle^2$ in addition to the term depending linearly on $(T_c - T)/T_c$ acquires a singular correction proportional to $\sqrt{(T_c - T)/T_c}$. At $T \rightarrow T_c$ this correction can catch up with the principal term. That renders an estimation of a region of “broadening” of the transition δT_{br} . At $(T_c - T) \ll \delta T_{br}$ temperature dependence of ρ_s and other thermodynamic properties of a superfluid is determined by fluctuations.¹² For Gaus-

sian fluctuations $\delta T_{br} \sim (T_c/N_\xi^2)(\xi_0/l)^4$, where N_ξ is a number of scattering centers within a volume with the radius $\sim \xi_0$. In aerogel the cross-section of a scattering center is $\sim 10^2$ times greater than that of an atomic impurity in a metal. It means that for the same mean free path number of centers in a unit volume is $\sim 10^2$ smaller. Coherence length ξ_0 for ^3He is of the same order as in a typical superconductor, so the fluctuations are bigger. Still for a 98% aerogel $N_\xi \approx 50$ and contribution of structural fluctuations dominates only in a region $(T_c - T)/T_c \sim 10^{-4} - 10^{-5}$, which is too narrow to explain the observed anomaly.

Porto and Parpia⁷ pointed to the fact that in aerogel the distribution of scattering centers is far from being random. They gave convincing experimental evidences of strong effect of structural correlations within the aerogel on the shift of the T_c and on the temperature dependence of ρ_s . Guided by these observations we consider theoretically effect of correlations in distribution of pair-breaking impurities on the shift of the T_c and on the temperature dependence of the square of the absolute value of the order parameter of a Fermi superfluid in a vicinity of T_c . We introduce in the argument of Ref. 11 effect of correlations. Because of the coherence of condensate its interaction with the ensemble of impurities depends on phase shifts between the perturbations brought about by the centers. The result of interference brings in the numerator of the expression for broadening of the transition an extra factor $\sim (nR^3)^2$, where R is the correlation radius. At sufficiently large R this factor compensates for N_ξ^2 in the denominator and effect of structural fluctuations can extend over an appreciable region below the T_c altering in this region temperature dependence of thermodynamic properties of the superfluid.

The paper is organized as follows: in Section II we consider correlated impurities in a superconductor with a scalar order parameter. Short account of these results was published before.^{13,14} Here we added as illustration an example of a system with a realistic correlation function and dropped all results of Ref. 13, which go beyond the perturbation theory and contain uncontrolled approximations. Reformulation of the argument for the p-wave superfluid ^3He is presented in Section III. In Section IV comparison with the NMR data for ABM and BW phases of ^3He is made.

II. SCALAR ORDER PARAMETER

In a vicinity of T_c effect of a quenched disorder can formally be described as a random spatial variation of coefficients in the Ginzburg and Landau equation. For a scalar order parameter Ψ :

$$a(\mathbf{r})\Psi + b(\mathbf{r})\Psi|\Psi|^2 - \nabla(c(\mathbf{r})\nabla\Psi) = 0. \quad (1)$$

Following the procedure of Ref. 11 we take into account only spatial dependence of $a(\mathbf{r})$, because it renders the

most singular contribution to the thermodynamic properties at $T \rightarrow T_c$. For $b(\mathbf{r})$ and $c(\mathbf{r})$ we use their average values $\langle b \rangle$ and $\langle c \rangle$. The coefficient $a(\mathbf{r})$ can be rewritten in terms of the local transition temperature $T_c(\mathbf{r})$: $a(\mathbf{r}) = \alpha(T - T_c(\mathbf{r}))$. It is convenient to divide $T_c(\mathbf{r})$ into its average value $\langle T_c \rangle$ and a relative fluctuation $\eta(\mathbf{r})$: $a(\mathbf{r}) = \alpha\langle T_c \rangle[\tau - \eta(\mathbf{r})]$, where $\tau = -1 + T/\langle T_c \rangle$ and $\eta(\mathbf{r}) = -1 + T_c(\mathbf{r})/\langle T_c \rangle$. After substitution $\Psi = \Psi_0\psi$ with $\Psi_0^2 = \alpha\langle T_c \rangle/\langle b \rangle$, and $\xi_s^2 = \langle c \rangle/\alpha\langle T_c \rangle$ Eq. (1) takes the form

$$[\tau - \eta(\mathbf{r})]\psi + \psi|\psi|^2 - \xi_s^2\Delta\psi = 0. \quad (2)$$

The random function $\eta(\mathbf{r})$ is treated as a perturbation. The global T_c is defined by the condition $\langle \psi \rangle \neq 0$ at $T < T_c$. Below T_c solution of Eq. (2) can be sought in a form $\psi(\mathbf{r}) = \langle \psi \rangle(1 + \chi(\mathbf{r}))$ and the perturbation procedure is justified when $\langle \chi^2(\mathbf{r}) \rangle \ll 1$. For real $\eta(\mathbf{r})$ essential part of χ is also real. Keeping in Eq. (2) terms up to the second order in χ and η we arrive at:

$$\tau - \eta + \tau\chi - \eta\chi + \langle \psi \rangle^2(1 + 3\chi + 3\chi^2) - \xi_s^2\Delta\chi = 0. \quad (3)$$

By the definition $\langle \eta(\mathbf{r}) \rangle = 0$ and $\langle \chi(\mathbf{r}) \rangle = 0$. Taking average of Eq. (3) we arrive at the equation for $\langle \psi \rangle^2$:

$$\langle \psi \rangle^2 = \frac{\langle \eta\chi \rangle - \tau}{1 + 3\langle \chi^2 \rangle}. \quad (4)$$

Subtraction of the averaged part from Eq. (3) renders linear equation for $\chi(\mathbf{r})$:

$$(\tau - \eta(\mathbf{r}) + 3\langle \psi \rangle^2)\chi - \xi_s^2\Delta\chi = \eta(\mathbf{r}) - \langle \eta\chi \rangle. \quad (5)$$

The second order term $3\langle \psi \rangle^2(\chi^2 - \langle \chi^2 \rangle)$ is neglected in comparison with $3\langle \psi \rangle^2\chi$. Equation (5) can be formally rewritten in terms of the Green function $G(\mathbf{r}, \mathbf{r}')$:

$$\chi(\mathbf{r}) = \int G(\mathbf{r}, \mathbf{r}')(\eta(\mathbf{r}') - \langle \eta\chi \rangle)d^3r'. \quad (6)$$

A straightforward argument shows that the average $\langle \eta\chi \rangle$ can be expressed via the self-energy of the averaged Green function of Eq. (5) in the momentum representation $\Sigma(\mathbf{k}, \tau)$:

$$\langle \chi\eta \rangle = \Sigma(0, \tau). \quad (7)$$

In a principal order on the perturbation $\eta(\mathbf{r})$

$$\Sigma(0, \tau) = \int \frac{\langle \eta(-\mathbf{k})\eta(\mathbf{k}) \rangle}{\tau + 3\langle \psi \rangle^2 - \Sigma(\mathbf{k}, \tau) + \xi_s^2 k^2} \frac{d^3k}{(2\pi)^3}. \quad (8)$$

Now we take $u = \Sigma(0, \tau) - \tau$ as a new variable. The self energy in the denominator of Green function may be expanded as $\Sigma(\mathbf{k}, \tau) \approx \Sigma(0, \tau) + \frac{\partial \Sigma}{\partial(k^2)}k^2$ and $\frac{\partial \Sigma}{\partial(k^2)}$ may be included in the definition of ξ_s^2 . Transition temperature is determined by the condition $u = 0$. In the second order on $\eta(\mathbf{r})$ it is

$$T_{c2} = \langle T_c \rangle [1 + \int \frac{\langle \eta(-\mathbf{k})\eta(\mathbf{k}) \rangle}{\xi_s^2 k^2} \frac{d^3k}{(2\pi)^3}]. \quad (9)$$

Using these definitions and Eq. (8) we arrive at the relation between u and $t = (T - T_{c2})/\langle T_c \rangle$:

$$u \left[1 + 2 \int \frac{\langle \eta(-\mathbf{k})\eta(\mathbf{k}) \rangle}{\xi_s^2 k^2 [2u + \xi_s^2 k^2]} \frac{d^3 k}{(2\pi)^3} \right] = -t. \quad (10)$$

In evaluation of the average $\langle \eta(-\mathbf{k})\eta(\mathbf{k}) \rangle$ we neglect effect of multiple scattering of quasiparticles by the scattering centers. That introduces a relative error in evaluation of $\langle T_c \rangle$ of the order of $x^{2/3}$, where x is the volume concentration of impurities. Assuming also that all scattering centers are identical we have: $\eta(\mathbf{r}) = \sum_a \eta^{(1)}(\mathbf{r} - \mathbf{r}_a) - \eta^{(1)}(0)$, where $\eta^{(1)}(\mathbf{r} - \mathbf{r}_a)$ is a local suppression of transition temperature by one center situated at the position \mathbf{r}_a , $\eta^{(1)}(\mathbf{k})$ - its Fourier transform, $\eta^{(1)}(0) \equiv \eta^{(1)}(\mathbf{k} = 0)$. Then $\langle \eta(-\mathbf{k})\eta(\mathbf{k}) \rangle = n|\eta^{(1)}(\mathbf{k})|^2 S(\mathbf{k})$, where the structure factor $S(\mathbf{k}) = \langle \sum_b \exp[i\mathbf{k}(\mathbf{r}_b - \mathbf{r}_a)] \rangle$ depends on the distribution of impurities. If impurities are not correlated all terms with $\mathbf{r}_b \neq \mathbf{r}_a$ for finite \mathbf{k} vanish at the averaging. Then $S(\mathbf{k}) = 1 + (2\pi)^3 n \delta(\mathbf{k})$. Here unity comes from the summand with $\mathbf{r}_b = \mathbf{r}_a$. The term proportional to $\delta(\mathbf{k})$ can be dropped because it eventually enters with the factor $\langle \eta(\mathbf{r}) \rangle = 0$. When correlations are present terms with $\mathbf{r}_b \neq \mathbf{r}_a$ render finite contribution to the structure factor. The number of additional terms in the sum is of the order of nR^3 , where R is correlation radius. If $nR^3 \gg 1$ their sum, which in what follows is denoted as $\tilde{S}(\mathbf{k})$ can be a principal contribution to $S(\mathbf{k})$. The $\tilde{S}(\mathbf{k})$ is directly related to the correlation function in the coordinate representation. For globally isotropic distributions the probability $w(\mathbf{r}_b|\mathbf{r}_a)$ to find a particle in the point \mathbf{r}_b if there is a particle in the point \mathbf{r}_a depends only on a distance $r = |\mathbf{r}_b - \mathbf{r}_a|$. At $r \rightarrow \infty$ correlations vanish and $w(r)$ tends to a constant. Normalization of $w(r)$ is usually chosen so that this constant is unity. Then a measure of correlations is $v(r) = w(r) - 1$. Changing in the definition of $S(\mathbf{k})$ summation for integration we arrive at

$$\tilde{S}(\mathbf{k}) = n \int v(r) e^{-i\mathbf{k}\mathbf{r}} d^3 r. \quad (11)$$

Eq. (10) can now be rewritten as

$$u[1 + W(u)] = -t, \quad (12)$$

where

$$W(u) = 2 \int \frac{n|\eta^{(1)}(\mathbf{k})|^2 [1 + \tilde{S}(\mathbf{k})]}{\xi_s^2 k^2 [2u + \xi_s^2 k^2]} \frac{d^3 k}{(2\pi)^3} \quad (13)$$

Numerator of the expression under the integral is the spectral density of fluctuations. It is multiplied by the response function, which is singular at $u \rightarrow 0$ and $k \rightarrow 0$. Contribution of fluctuations with $k \sim 1/\xi(u)$, where $\xi(u) = \xi_s/\sqrt{2u} \gg \xi_s$ is strongly enhanced. The perturbation $\eta^{(1)}(\mathbf{r})$ falls out on a distance $\sim \xi_s$ from the center so that in the essential region of k $\eta^{(1)}(\mathbf{k}) \approx \eta^{(1)}(0) \sim \xi_s \sigma$, where σ is a cross-section of scattering of a quasi-particle by one center. When fluctuations are not correlated

$\tilde{S}(\mathbf{k}) = 0$, the spectral density is constant and we recover the result of Larkin and Ovchinnikov¹¹ for broadening of the transition: $W_1(u) = n|\eta^{(1)}(0)|^2/(2\pi\xi_s^3\sqrt{2u})$. Rough estimation renders $W_1(u) \sim (\xi_s/l)^2 \frac{1}{n\xi_s^3\sqrt{u}}$. For ^3He in the 98% aerogel $W_1(u) \ll 1$ if $u \geq 10^{-4}$. Contribution of correlations

$$W_2(u) = 2n|\eta^{(1)}(0)|^2 \int \frac{\tilde{S}(\mathbf{k})}{\xi_s^2 k^2 [2u + \xi_s^2 k^2]} \frac{d^3 k}{(2\pi)^3}. \quad (14)$$

depends on the behavior of $\tilde{S}(\mathbf{k})$ at $k \sim 1/\xi(u)$. X-rays scattering data^{7,8,15} show that silica aerogels have fractal structure with the fractal dimension $D_f \approx 1.7 \div 1.9$ depending on a sample. Formally it means that at $k \rightarrow 0$ the $\tilde{S}(\mathbf{k})$ grows as $1/k_f^{D_f}$ increasing density of the long wavelength fluctuations until it saturates at $k \sim 1/R$. Further enhancement comes from the response function. It is characterized by the temperature dependent coherence length $\xi(u)$. If $R \gg \xi_s$ there are two regions of u , corresponding to different asymptotic behavior of $W_2(u)$: $\xi(u) > R$ and $\xi(u) < R$ with different asymptotic. For finding the asymptotic we express $W_2(u)$ in terms of the correlation function $v(r)$:

$$W_2(u) = \frac{n^2|\eta^{(1)}(0)|^2}{\xi_s^2 u} \int [1 - \exp(-r/\xi(u))] v(r) r dr. \quad (15)$$

In a simplest case $v(r)$ can be characterized by two parameters - a correlation radius R and an overall amplitude A . Convenient model form was suggested in Ref. 15:

$$v(r) = A \left(\frac{R}{r} \right)^{3-D_f} \exp(-r/R). \quad (16)$$

This expression reproduces qualitative features of correlations in silica aerogels i.e. the existence of a macroscopic correlation radius R and a fractal behavior with dimensionality D_f at $r < R$. Introduction of the dimensionless variable $y = r/R$ renders:

$$W_2(u) = \frac{n^2 R^2 |\eta^{(1)}(0)|^2}{\xi_s^2 u} \int [1 - \exp(-yR/\xi(u))] v(y) y dy. \quad (17)$$

Asymptotic of this integral at $(R/\xi(u)) \rightarrow 0$ is $W_2(u) \approx (n^2|\eta^{(1)}(0)|^2 R^3 \sqrt{2}/\sqrt{u}\xi_s^3) \int_0^\infty v(y) y^2 dy$. It has the same $1/\sqrt{u}$ singularity at $u \rightarrow 0$ as the contribution of non-correlated impurities. Its sign and the relative weight depend on the value of the integral $I_v = \int_0^\infty v(y) y^2 dy \sim A$. It means that the contribution of correlations has an extra coefficient AnR^3 . For realistic values of parameters effect of fluctuations in aerogel is enhanced.

In the opposite limit $R \gg \xi(u)$ the exponent in the square bracket under the integral in Eq. (15) can be neglected and $W(u) \approx \frac{nR^2}{2\xi_s^2 u} \int_0^\infty v(y) y dy$. That renders a constant shift in the relation between u and t which is equal to $(T_{c2} - \langle T_c \rangle)/\langle T_c \rangle$ so that eventually $u \approx -\tau$.

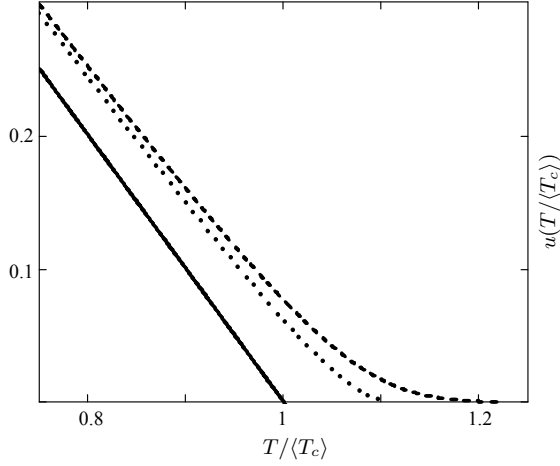


FIG. 1. Dependence u on $T/\langle T_c \rangle$, as given by Eqns. (12) - (16) for different ratios of R/ξ_s at a fixed value of the product $A(R/\rho)^{3-D_f}$. Solid line corresponds to non-correlated case, i.e. $R/\xi_s = 0$. Dotted line corresponds to $R/\xi_s = 3$ and dashed line is for $R/\xi_s = 7$.

This is a straight line, which extrapolates to $T = \langle T_c \rangle$ at $u = 0$.

Relation (12) between u and t was derived with the use of perturbation theory, it breaks down when $W(u) \sim 1$. This condition determines a region of broadening of the transition by structural disorder. Using the asymptotic of $W(u)$ at $u \rightarrow 0$ we arrive at the estimation of the width of this region: $u \leq (AR^3/\xi_s l^2)^2$. The estimation coincides with one obtained from the condition $\langle \chi^2 \rangle \ll 1$. It follows from the asymptotic at $R \ll \xi(u)$ of the expression

$$\langle \chi^2 \rangle = \int \frac{n|\eta^{(1)}(\mathbf{k})|^2[1 + \tilde{S}(\mathbf{k})]}{[2u + \xi_s^2 k^2]^2} \frac{d^3 k}{(2\pi)^3}. \quad (18)$$

The region of broadening of the transition for correlated impurities is $(AnR^3)^2$ times more wide than for non-correlated and the “tail” of the singular correction to the linear dependence of u on t extends for a larger interval of u into the region $(AR^3/\xi_s l^2)^2 \ll u \ll 1$ where perturbation theory holds. If $R \sim \xi_s$ the deviations of $\langle \psi \rangle^2$ from the linear law can persist in a region well below the transition temperature in agreement with the data for superfluid ^3He in aerogel.^{6,7}

For illustration of qualitative changes, introduced by correlations we plot on the Fig. 1 the dependence $u(T/\langle T_c \rangle)$, given by Eqns. (12) - (16) for different values of parameters, characterizing aerogel. As one can see the transition temperature shifts up with respect to $\langle T_c \rangle$ when R increases. Relative value of the shift according to Eq. (9) is of the order of $(R/l)^2$. Ratio of the positive shift to $T_{cb} - \langle T_c \rangle$ is of the order of $R^2/\xi_s l$. If $R \gg \xi_s$ a major change of the slope of $u(t)$ takes place at $u \sim (\xi_s/R)^2$.

Function $u(t)$ determines also correction to the temperature dependence of the specific heat below T_{ca} . Difference of free energies of the superfluid and normal phases

can be represented as:

$$F_s - F_n = -\frac{\Delta C_b}{2T_{cb}} \langle T_c \rangle^2 \int \langle |\psi|^4 \rangle d^3 r, \quad (19)$$

where ΔC_b is the specific heat jump in pure ^3He . With account of the terms of the order of $\langle \chi \rangle^2 \langle |\psi|^4 \rangle = u^2$, then:

$$\frac{1}{T}(C_s - C_n) = \frac{\Delta C_b}{2T_{cb}} \frac{\partial^2 (u^2)}{\partial t^2}. \quad (20)$$

For $u(t)$ given by Eqns. (12), (13) this expression remains finite up to T_{ca} , but its validity is justified only in a region where the condition $\langle \chi^2 \rangle \ll 1$ is met.

III. P-WAVE PAIRING

For application to the superfluid ^3He in aerogel the argument of Section II has to be reformulated for the p-wave Cooper pairing. The order parameter in that case is 3×3 complex matrix $A_{\mu j}$. The first index refers to 3 spin components, and the second – to 3 components of the angular momentum of Cooper pairs. Aerogel interacts with the orbital part of the order parameter. This interaction is described by the additional term in the density of free energy:

$$f_\eta = -N_{eff}(\eta_{jl}(\mathbf{r}) + \kappa_{jl})A_{\mu j}A_{\mu l}^*. \quad (21)$$

Now $\eta_{jl}(\mathbf{r})$ is a real and symmetric 3×3 random tensor of local anisotropy. The ensemble average $\langle \eta_{jl} \rangle = 0$. The \mathbf{r} -independent tensor κ_{jl} is introduced for account of possible global anisotropy. In the present calculations a uniaxial anisotropy will be assumed only implicitly for lifting of the orientational degeneracy of the A-like phase. Within the used approximation the global anisotropy will not enter results and it will be dropped from the free energy. N_{eff} is the overall factor, having dimensionality of density of states, it does not enter the resulting equations. For the gradient energy we take a simplified isotropic expression:

$$f_\nabla = N_{eff}\xi_s^2 \left(\frac{\partial A_{\mu l}}{\partial x_n} \frac{\partial A_{\mu l}^*}{\partial x_n} \right). \quad (22)$$

Both terms (21) and (22) are added to the unperturbed density of GL free energy:

$$f_0 = N_{eff}(\tau A_{\mu j}A_{\mu j}^* + \frac{1}{2} \sum_{s=1}^5 \beta_s I_s) \quad (23)$$

where I_s - are the 4-th order invariants in the expansion of the free energy over $A_{\mu j}$:¹⁶ $I_1 = A_{\mu j}A_{\mu j}^*A_{\nu l}^*A_{\nu l}$, $I_2 = A_{\mu j}A_{\mu j}^*A_{\nu l}A_{\nu l}^*$, $I_3 = A_{\mu j}A_{\nu j}A_{\mu l}^*A_{\nu l}^*$, $I_4 = A_{\mu j}A_{\nu j}^*A_{\nu l}A_{\mu l}^*$, $I_5 = A_{\mu j}A_{\nu j}^*A_{\mu l}A_{\nu l}^*$, and β_1, \dots, β_5 are phenomenological coefficients. The resulting GL equations have the form:

$$\frac{\partial f_0}{\partial A_{\mu j}^*} - \xi_s^2 \left(\frac{\partial^2 A_{\mu j}}{\partial x_n^2} \right) = -A_{\mu l} \eta_{jl}(\mathbf{r}). \quad (24)$$

To follow the perturbation procedure of the previous section we split the order parameter into its average and the fluctuation. In the B-like phase we search for a solution in the form $A_{\mu j} = \langle A_{\mu l} \rangle (\delta_{lj} + c_{lj}(\mathbf{r}))$. Here $\langle A_{\mu j} \rangle = \bar{\Delta} R_{\mu j}$, $R_{\mu j}$ is a real orthogonal matrix and $c_{lj}(\mathbf{r})$ is real and symmetric. Procedure analogous to that used at the derivation of Eq. (4) renders an equation for $\bar{\Delta}^2$:

$$\bar{\Delta}^2 \{ (3\beta_{12} + \beta_{345}) + \beta_{12345} \langle c_{ml} c_{lm} \rangle + \frac{2}{3} \beta_{12} \langle c_{mm} c_{ll} \rangle \} = \frac{1}{3} \langle \eta_{ml} c_{lm} \rangle - \tau. \quad (25)$$

Here $\beta_{12} = \beta_1 + \beta_2$ etc.. Tensor of fluctuations c_{jl} satisfies the linear equation:

$$(\tau \delta_{jn} - \eta_{jn}) c_{nl} - \xi_s^2 \nabla^2 c_{jl} + \bar{\Delta}^2 [3\beta_{12345} c_{jl} + \beta_{12} 2c_{nn} \delta_{jl}] = \eta_{jl} - \langle \eta_{jn} c_{nl} \rangle. \quad (26)$$

As before we consider aerogel as a random distribution of identical scattering centers. The center situated at a point \mathbf{r}_a introduces a local perturbation $\eta_{jl}^{(1)}(\mathbf{r} - \mathbf{r}_a)$. For sufficiently dilute system, when effect of multiple scattering of quasi-particles by impurities can be neglected, the overall perturbation is a sum of contributions of individual impurities: $\eta_{jl}(\mathbf{r}) = \sum_a \eta_{jl}^{(1)}(\mathbf{r} - \mathbf{r}_a)$. Its Fourier transform is:

$$\eta_{jl}(\mathbf{k}) = \eta_{jl}^{(1)}(\mathbf{k}) \sum_a \exp(-i\mathbf{k}\mathbf{r}_a). \quad (27)$$

As has been discussed in Section II most essential are fluctuations with small \mathbf{k} , which meet the condition $k\xi_s \ll 1$. That makes possible to substitute for $\eta_{jl}^{(1)}(\mathbf{k})$ its limiting value at $\mathbf{k} \rightarrow 0$ which is $\delta_{jl}\eta^{(1)}(0)$, leaving dependence on \mathbf{k} only in the phase factors $\exp(-i\mathbf{k}\mathbf{r}_a)$. In the coordinate representation it means that the scattering centers are considered as isotropic and point-like $\eta_{jl}^{(1)}(\mathbf{r}) = \delta_{jl}\eta^{(1)}(0)\delta(\mathbf{r} - \mathbf{r}_a)$. The total tensor $\eta_{jl}(\mathbf{r})$ in that case is also proportional to δ_{jl} . By the argument of Rainer and Vuorio¹⁷ $\eta^{(1)}(0) = \gamma(\xi_s/nl)$ where l is a transport mean free path and the coefficient γ depends on properties of the centers. For diffusely scattering balls $\gamma = \pi^2/4$.¹³ Symmetric tensor c_{jl} in Eq. (26) can be represented as a sum of its scalar $c_{nn}\delta_{jl}$ and traceless $c_{jl}^{(s)} = \frac{1}{2}(c_{jl} + c_{lj} - \frac{2}{3}c_{nn}\delta_{jl})$ parts. The system of equations (26) splits then in two independent systems for each part of c_{jl} . Of the two only equation for the scalar part contains a finite perturbation in the right hand side, so that the solution can be sought in a form $c_{jl} = \delta_{jl}\chi$. Substitution of $\eta_{jl} = \eta\delta_{jl}$ in Eqns. (25), (26) and introduction of the notation $\langle \psi \rangle^2 = \bar{\Delta}^2(3\beta_{12} + \beta_{345})$ transforms Eq. (25) into Eq. (4) and Eq. (26) – into Eq. (5). The problem for ³He-B turns out to be equivalent to that for the scalar order parameter.

In the A-like phase situation is complicated by a possibility of formation of the disordered Larkin-Imry-Ma (LIM) state. Global anisotropy orients the order parameter and restores the long-range order. Experiments¹⁸ and theoretical estimations^{19,20} show that compression

of aerogel for a few percent along one direction is sufficient for the restoration of the long range order. In this case reduction to the scalar problem is possible for the A-like phase as well. In the bulk liquid the order parameter of the A-phase is a direct product of spin and orbital vectors: $A_{\mu j} = d_{\mu} A_j$. Aerogel coated by ⁴He does not interact with the spin part of the order parameter. We seek a solution of Eq. (24) also in a separable form, then it reduces to the equation for the orbital vector A_j :

$$[\tau \delta_{jl} - \eta_{jl}(\mathbf{r})] A_l - \xi_s^2 \left(\frac{\partial^2 A_j}{\partial x_n^2} \right) + \beta_{13} A_j^* (A_s A_s) + \beta_{245} A_j (A_s A_s^*) = 0 \quad (28)$$

In line with the discussion in the paragraph following Eq. (27) we keep only isotropic part of the perturbation $\eta_{jl}(\mathbf{r}) = \delta_{jl}\eta(\mathbf{r})$ and seek the solution in a form $A_l = \langle A_l \rangle + a_l$. Here $\langle A_l \rangle = (1/\sqrt{2})\bar{\Delta}(\hat{m} + i\hat{n})$, unit vectors \hat{m} and \hat{n} are mutually orthogonal and orthogonal to the direction of compression. For definiteness we have chosen one of two possible orientations of $\langle A_l \rangle$. Repeating the procedure of Section II we find as the solution $a_l = \chi(\mathbf{r})\langle A_l \rangle$. Denoting $\beta_{245}\bar{\Delta}^2 = \langle \psi \rangle^2$ we return to the equations (4) and (5) of the scalar problem.

The reduction to the scalar order parameter is based on the fact that principal contribution to thermodynamic anomalies comes from inhomogeneities with small wave vectors. Such inhomogeneities interact with fluctuations of the amplitude of the order parameter, their coupling with the other collective modes of B-like and A-like phases appears only in the next order on u .

IV. COMPARISON WITH EXPERIMENT

The obtained expressions allow comparison with experiment. Properties of aerogel enter these expressions via the mean free path l and the structure factor $S(\mathbf{k})$. Both quantities in principle can be found from independent experiments, l – from the measurement of spin diffusion in the normal phase,²¹ and $S(\mathbf{k})$ – from the X-ray scattering data. Physical quantities which can be compared with our calculations are e.g. the temperature dependence of the square of the longitudinal resonance frequency Ω_L^2 , which in a vicinity of T_c is proportional to $\bar{\Delta}^2$ i.e. $\langle \psi \rangle^2$ (Eqns. (4), (12), (13)) and the lowering of the transition temperature, given by Eq. (9). In realization of the comparison we encounter two difficulties. Firstly the thermodynamic data suitable for the comparison are not accompanied by the independently measured l and $S(\mathbf{k})$ for the same sample of aerogel. Even when $S(\mathbf{k})$ is measured the result is given in arbitrary units, leaving ambiguity in the overall amplitude. Secondly the theoretical expressions are obtained within the perturbation theory and their application is limited by a region of small fluctuations. Since the discussed effects originate from fluctuations they are small in a region of applicability of theory. So, we have to find a compromise between the

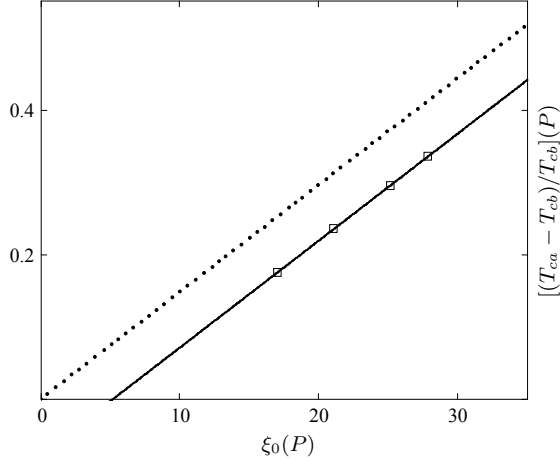


FIG. 2. Fitting of the dependence of lowering of the transition temperature of ^3He in aerogel (data points from Ref. 6) as a function of $\xi_0 = 2\pi\hbar v_F/T_{cb}$ by the straight line $\gamma\xi_0/l + \delta$ (solid line). The slope $\gamma = 2.95$ falls between its values for specular $\gamma_{sp} = \pi^2/3$ and diffuse scattering $\gamma_{dif} = \pi^2/4$. The constant shift $\delta = 0.075$ is a contribution of correlations to the change of the transition temperature ($\delta \sim AR^2/l^2$). The dotted line corresponds to non-correlated impurities.

magnitude of the effect and accuracy of its description. With these reservations we choose for comparison a temperature dependence of Ω_L^2 , extracted from the measurements of the transverse NMR frequency shift.²² In these experiments neither l nor $S(\mathbf{k})$ was measured, but the data was taken for several pressures with the same sample of 98% aerogel and in a suitable temperature interval. For such porosity l is roughly estimated as 130-180 nm.²¹ Fine tuning of l within this interval is made in a process of fitting. Unknown structure factor formally introduces infinitely many fitting parameters, but in all equations $S(\mathbf{k})$ enters under the sign of integral. The main contribution to the integral in Eq. (13) comes from the region of small \mathbf{k} so that only the asymptotic form of $S(\mathbf{k})$ at $k \rightarrow 0$ or of $v(r)$ at $r \rightarrow \infty$ is essential. The integral in Eq. (9) for $D_f > 1$ is also defined by a region of small \mathbf{k} . The answers are not very sensitive to the detailed form of $S(\mathbf{k})$ or of $v(r)$, so we use for interpretation of data the model expression Eq. (16) with two fitting parameters A and R . Lowering of the transition temperature by aerogel was discussed before.¹³ In the leading order on ξ_0/l the principal contribution is proportional to this ratio and correction due to correlations depends on the combination AR^2/l^2 . For comparison we use the data of Ref. 22 taken at four pressures: 29.3, 17.5, 20.1 and 11.9 bar. Transition temperatures for all four pressures can be fitted by a straight line as shown on the Fig. 2 and the combination AR^2/l^2 can be extracted from the fit. Additional combinations of parameters are found by fitting temperature dependencies of Ω_L^2 in the B-like phase, as shown on the Fig. 3. The procedure requires further fitting parameters originating from the phenomenological coefficients, entering Eqns. (1), (24). The amplitude

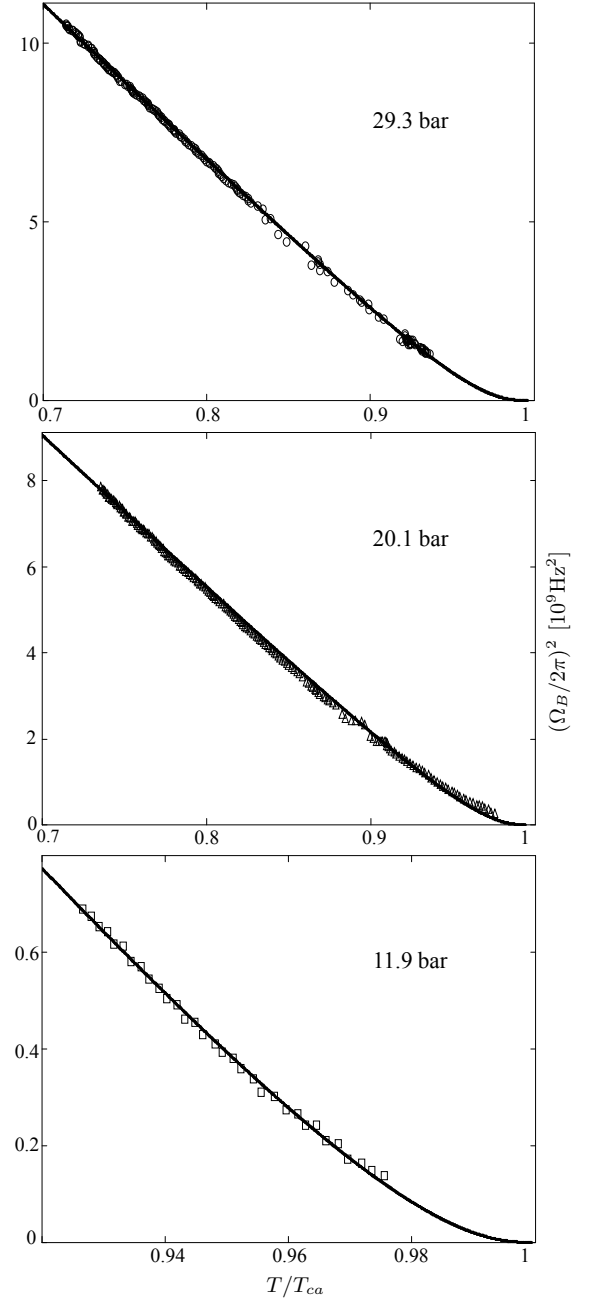


FIG. 3. Temperature dependence of the square of the longitudinal resonance frequency Ω_B^2 in the B-like phase at three different pressures. Data points are taken from Ref. 6, solid lines – results of fitting. Parameters of aerogel are $l = 140$ nm, $R = 35$ nm, $A = 0.055$. Coherence length is taken as $\xi_s = 0.71\xi_0$.

Ψ_0^2 has to be fitted for each pressure. About the coherence length ξ_s we assume that it scales as $(\hbar v_F)/T_{cb}$ and fit one coefficient for all pressures. Parameters of aerogel, which provide the best fit for the lowering of the transition temperature for all four pressures and for the temperature dependence of the square of the longitudinal resonance frequency Ω_L^2 for three pressures, shown

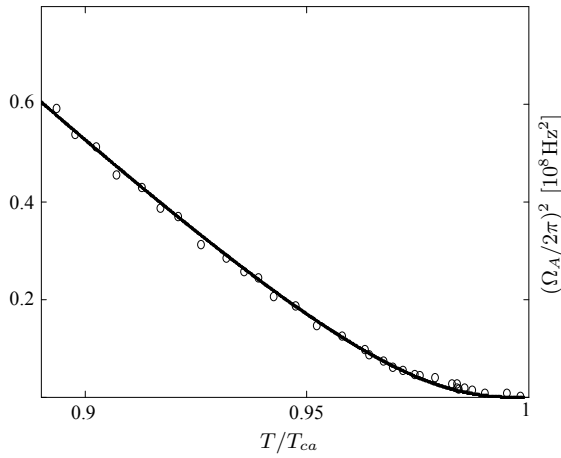


FIG. 4. Temperature dependence of the square of the longitudinal resonance frequency Ω_A^2 in the A-like phase at $P=29.3$ bar. Data points are taken from Ref. 22, solid line – results of fitting with parameters of aerogel – $l = 140$ nm, $R = 35$ nm, $A = 0.055$. Coherence length is taken as $\xi_s = 0.71\xi_0$.

on the Fig. 2 are the following: $l = 140$ nm, $R = 35$ nm, $A = 0.055$.

Part of the data fall in a region where $\langle\chi\rangle^2$ is not very small. Estimation, made with the cited parameters renders for $P=11.4$ bar $\langle\chi\rangle^2 \approx 0.04/\sqrt{u}$ at $u \rightarrow 0$. At the position of the closest to T_c experimental point $\langle\chi\rangle^2 \approx 1/3$. Fluctuations are even bigger at $P=20.1$ bar, where $\langle\chi\rangle^2 \approx 0.1/\sqrt{u}$. Argument, based on the perturbation theory can not apply in a region of developed fluctuations. The argument certainly breaks down when $\langle\chi\rangle^2 \approx 1$. For $P=20.1$ bar it would correspond to $u = 0.01$.

For the A-like phase we also use the NMR data of Ref. 22, taken in a condition when compression is sufficiently strong for orientation of the average order parameter. Fig. 4 demonstrates that a satisfactory agreement with experiment can be achieved.

The results of comparison show that the data for both A-like and B-like phases can be fitted with the values of parameters which are within the range admitted by other experiments with different samples of 98% aerogel. That provides a support to the suggested interpretation of the

observed anomalies of thermodynamic properties of the superfluid ^3He in aerogel.

V. CONCLUSION

When discussing reaction of a superconductor or a superfluid on a quenched disorder it is important to take into consideration that reaction of the condensate of Cooper pairs increases with a decrease of a wave vector of a perturbation \mathbf{k} , saturating at $k \sim 1/\xi(u)$. Correlations between impurities influence distribution of fluctuations over their Fourier spectra. If, like in aerogel, correlations increase the weight of the long wavelength fluctuations effect of the disorder can be significantly increased in comparison with the uniform spectral distribution of the same impurities. This results in the increase of a temperature interval below T_c where fluctuations dominate. The fluctuation region has a “tail”, in which fluctuations of the order parameter are sufficiently small to be treated by the perturbation theory, but their effect on thermodynamic properties of the superfluid is still distinguishable. A length of the tail depends on the relation between the coherence length of the condensate ξ_0 and correlation radius R . For $R \approx \xi_0$ a trace of fluctuations extends for the most of the Ginzburg and Landau region.

Shift of fluctuations in a region of small k enhances the transition temperature of ^3He (cf. Eq. (9)). Analogous effect was discussed before as a way to further enhance the T_c in high temperature superconductors by ordering of impurities or modulation of pairing interaction.^{23,24} Advantage of ^3He in aerogel as an object of investigation is that aerogel forms a rigid structure, which can be studied separately from ^3He .

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